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THE NUMERICAL EVALUATION OF A PHYSICAL OPTICS NORMALIZED CROSS SECTION FOR A ROUGH SURFACE

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One major topic in this report is the explanation of the numerical evaluation of the physical optics normalized cross section (σ^0) of a one-dimensional rough surface. A second topic involves the comparison of the numerical evaluation of σ^0 with two other approximate evaluations of the physical optics integral representation for σ^0. Some comparisons of these three representations for σ^0 are made as a function of surface roughness and mean surface slope. A general integral expression for σ^0 for a one-dimensional rough surface is analyzed and it is shown how it may be evaluated numerically. The single integral approximation is shown to require small surface slopes for good agreement whereas the geometric optics solution requires both small slopes and large Rayleigh parameter. Keywords:

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Preface

This report represents the combined results of the authors individual investigations. Dr. Papa formulated the theory and analyzed the results (Section 1, 2, 3, and 6) while Mrs. Woodworth developed the mathematical techniques (described in Section 4) that enabled the numerical results to be obtained (Section 5).

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The Numerical Evaluation of a Physical Optics Normalized Cross Section for a Rough Surface

1. INTRODUCTION

This report is devoted to two main topics: The detailed explanation of the numerical evaluation of the physical optics normalized cross section of a rough surface (σ°) and the comparison of these numerical results with two other approximate evaluations of the physical optics integral representation for σ°. In a previous paper[1], the general integral expression for the normalized cross section σ° was given for a surface which is randomly rough in one dimension only and some preliminary comparisons of the three representations for σ° were made as a function of surface roughness and mean surface slope. In this report, we analyze this general integral expression for σ° and show how it may be evaluated numerically. Additional comparisons are included.

The basic condition for a physical optics (PO) model to be valid is that $R_c >> \lambda$ where R_c is the average radius of curvature of the surface and λ is the em wavelength. In previous work[2], we showed that the condition $T >> \lambda$ (T is the surface correlation length) is a sufficient condition for $R_c >> \lambda$ if the surface slopes are small, but it becomes necessary as well as sufficient if larger slopes are not excluded. That demonstration assumed Gaussian heights and correlation. In this paper, we will be concerned

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1. Papa, R.J. and Lennon, J.F. (1988) Regions of Validity for Some Rough Surface Scattering Models, "Scattering and Propagation in Random Media", AGARD Conference Proceedings, 419.

2. Papa, P.J., Lennon, J.F., and Taylor, R.L. (1984) *An Analysis of Physical Optics Models for Rough Surface Scattering*, RADC-TR-84-195, ADA154960.

mainly with small slope conditions. Under the assumption $R_c >> \lambda$, Barrick and Peake[3] have given a general expression for a PO σ° with no shadowing. This expression generally[4-7] has been evaluated by taking the high frequency limit, $k = 2\pi/\lambda \rightarrow \infty$ (GO) which allows decorrelation of the ensemble averaging over the height distribution from that of the surface slope distribution followed by an asymptotic algebraic result. It has been shown[1] that their product form representation:

$$\sigma^\circ = \mid \beta_{pq} \mid^2 J \tag{1}$$

is valid without the restriction $\lambda \rightarrow 0$, so long as the surface slopes are small ($\sigma/T < 1$) and J is kept as an integral form. Here, β_{pq} is the scattering matrix element (8), p refers to the polarization of the incident wave and q refers to the polarization of the scattered wave (horizontal or vertical), J is proportional to the probability density function of the surface slopes and σ is the standard deviation in surface height. Gaussian heights and correlation are again assumed. This result and the high frequency form are then compared with numerical evaluation of the general expression for σ° for forward scattering and horizontal polarization.

The conventional expression for the normalized cross section σ° may be defined by applying physical optics principles to rough surface scattering, where the Kirchhoff integral is used to represent the scattered em wave and the boundary conditions on the surface are satisfied by employing the Fresnel plane wave reflection coefficients[1-8]. Multiple scattering is not included. Following these procedures the Barrick and Peake[3] generalized expression for the field scattered from a rough surface is (horizontal polarization):

$$E^{inc} = -ik[e^{ikR_o}/4\pi R_o]E_h^i \int_{-L/2}^{L/2}\int_{-L/2}^{L/2} \mathbf{F}(\xi_x, \xi_y)[e^{ik(\mathbf{k}_i - \mathbf{k}_s)\cdot \mathbf{r}}]dxdy \tag{2}$$

where

E_h^i = incident, horizotally polarized electric field,

R_o = distance from origin to observation point,

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3. Barrick, D.E. and Peake, W.H. (1967) *Scattering from Surfaces with Different Roughness Scales: Analysis and Interpretation*, Battelle Report, AD662751.
4. Semyonov, B.I. (1966) Approximate computation of scattering of electromagnetic waves by rough surface contours, *Radiotekhnika i Electronika* **11**, 1179-1187.
5. Barrick, D.E. (1968) Rough surface scattering based on the specular point theory, *IEEE Trans. Antennas Propag.* **AP-16**(4), 449–454.
6. Kodis, R. (1966) A note on the theory of scattering from an irregular surface, *IEEE Trans. Antennas Propag.* **AP-14**(1), 77–82.
7. Sancer, M.I. (1969) Shadow corrected electromagnetic scattering from a randomly rough surface, *IEEE Trans. Antennas Propag.* **AP-17**(5), 575-585.
8. Ruck, G.T., Barrick, D.E., Stuart, W.D., and Krichbaum, C.K. (1970) *Radar Cross Section Handbook, Vol.* **2**, New York, Plenum Press.

$\mathbf{r} = x\mathbf{x} + y\mathbf{y} + \xi(x,y)\mathbf{z}$ = distance from origin to point on rough surface,

$\mathbf{k}_i$, $\mathbf{k}_s$ = unit constant vectors pointing in direction of incidence and scattering,

ξ_x, ξ_y = local surface slopes in x and y directions at surface point

$\xi(x,y)$, that is, $\partial\xi/\partial x$ and $\partial\xi/\partial y$,

The factor $\mathbf{F}(\xi_x, \xi_y)$ is a function of the local normal to the surface and of local Fresnel reflection coefficients at each surface point. The Barrick and Peake expression for $\mathbf{F}$ is incorrect as printed. The correct form is given by Sancer[7].

To form the normalized cross section of the rough surface, $\sigma°$, it is necessary to calculate

$$\sigma° \ \alpha \ \langle \mathbf{E}^{s*} \cdot \mathbf{E}^{s} \rangle - \langle \mathbf{E}^{s} \rangle^2 \tag{3}$$

where $\langle \cdot \rangle$ denotes an ensemble average over the random variables ξ, ξ_1, ξ_x, ξ_y, ξ_{x1}, and ξ_{y1}. Here, ξ is the random hight at point (x, y) and ξ_1 is the random height at point (x_1, y_1). The general PO expression [Eq. (3)] for $\sigma°$ involves a ten-fold integration over the variables x, y, x_1, y_1, ξ, ξ_1, ξ_x, ξ_y, ξ_{x1}, and ξ_{y1}. By making use of the stationarity of the random process, this expression can be reduced to an eight-fold integral. By what is traditionally referred to as a geometrical optics assumption, this result is further reduced to a single integral form and finally evaluated as an algebraic expression.. The single integral form can also be obtained by making use of a small slope condition[1]. For that case, though, the final evaluation does not apply.

2. THEORY FOR SCATTERING FROM A ONE-DIMENSIONAL ROUGH SURFACE

To simplify the general expression for $\sigma°$ and to make numerical evaluations of $\sigma°$ more efficient, it will be assumed that the random distribution in heights, ξ, have only a one-dimensional variation $\xi = \xi(x)$. It will also be assumed that the scattering takes place in the direct forward direction, so that there is no azimuthal variation ($\varphi_s = 0°$) and that there is no shadowing. The most general expression for $\sigma°$ for a one-dimensional rough surface involves a six-fold integral, where the variables of integration are x_1, x_2, ξ_1, ξ_2, μ_1, and μ_2. Here, x_1 and x_2, are two points on the rough surface, ξ_1 and ξ_2 are the random surface heights at the two points and μ_1 and μ_2 are the random surface slopes at the two points. By using the fact that the surface heights are to be regarded as a stationary random process, so that the correlation function of the surface heights, $c(\tau)$, is a function only of the separation between points $\tau = x_1 - x_2$, the six-fold integral for $\sigma°$ may be reduced to a five-fold integral. Then, using the fact that the general expression for $\sigma°$ is a function only of the height differences $\xi = \xi_1 - \xi_2$, the five-fold integral can be reduced to a four-fold integral. Finally, by assuming the trivariate distribution function for the heights ξ and slopes μ_1 and μ_1 is a Gaussian function, the integration over ξ can be performed analytically. Then, the remaining expression for $\sigma°$ reduces to a triple integral, which can be performed numerically using Gaussian quadrature techniques.

Following the development given by Hagfors[9], the expression for σ° may be written as (1):

$$\sigma^\circ = 2\pi k \int_0^\infty d\tau \;\cos v_x\tau \;\; w(\tau) \tag{4}$$

where

$v_x = k(\sin\theta_i - \sin\theta_s)$

$\tau = x_1 - x_2$ (separation between two points on the rough surface)

and

$$w(\tau) = G(\tau) - H. \tag{5}$$

$$G(\tau) = 2 \int_{-\infty}^{\infty} d\mu_2 F(\mu_2) \int_{-\infty}^{\infty} d\mu_1 F(\mu_1) \int_0^\infty d\xi \;\cos v_z \xi \cdot P_3(\mu_1, \mu_2, \xi, \tau)$$

P_3 = trivariate distribution function in height differences and surface slopes. In addition, ξ_1, μ_1, and ξ_2, μ_2 are the heights and slopes of point 1 and point 2, respectively, with $\xi = \xi_1 - \xi_2$, $\mu_1 = \xi_{x1}$ and $\mu_2 = \xi_{x2}$.

$F(\mu)$ is a complicated function of the slopes μ, and is given in the appendix, as is the form for the quantity H. It can be shown that $w(\tau) \to 0$ as $\tau \to \infty$.

There are no restrictions on the expression for σ° given by [Eq. (4)], other than the validity of physical optics, $T >> \lambda$. There are singularities in the integrand of [Eq. (4)] but they can be shown to be integrable and the expression has been evaluated numerically, using quadrature formulas to provide a standard for the two limiting case solutions. For the two sets of assumptions considered here, the expressions in [Eq. (4)] and [Eq. (5)] reduced to the simpler form of [Eq. (6)] involving a single integration over τ:

$$\sigma^\circ = 2\pi k \;\; F^2(u_{sp}) \int_0^\infty d\tau \;\cos v_x\tau \cdot [\chi_2 - \chi_1^2] \tag{6}$$

where

$\chi_2 = \exp[-\sigma^2 v_z^2(1 - \rho)]$

$\rho = \exp[-\tau^2/T^2]$

$\chi_1 = \exp(-v_z^2\sigma^2/2)$

9. Hagfors, T. (1964) Backscattering from an undulating surface with applications to radar returns from the moon, *J. Geophysical Res.* **69**(18), 3779-3784.

and

$$v_z = -k(\cos\theta_i + \cos\theta_s).$$

The details of the arguments in each case are presented in the appendix. Finally, for the case where $\lambda \to 0$ and $\Sigma > 1$, the integral of [Eq. (6)] can be explicitly evaluated for σ° using the stationary phase method[7]:

$$\sigma^\circ = \pi k F^2(\mu_{sp})(T\sqrt{\pi}/\Sigma)\exp(-v_x^2 T^2/4\Sigma^2) \tag{7}$$

Here, $\Sigma = -\sigma v_z$

One of our objectives here is to examine the conditions under which each of these two forms agrees with a direct numerical evaluation of [Eq. (4)] and [Eq. (5)] and to define the regions where one or the other limiting case solutions would be preferred. These are discussed in Section 5.

3. SIMPLIFICATION OF EXPRESSION FOR NORMALIZED CROSS SECTION

[Eqs. (4) and (5)] constitute exact physical optics expressions for the normalized cross section σ° of a one dimensional rough surface. These equations show that this general expression for σ° involves a four-fold integration over the variables τ, ξ, μ_1 and μ_2. If the trivariate distribution function $P_3(\xi, \mu_1, \mu_2)$ in ξ, μ_1 and μ_2 is assumed to be a Gaussian, then the integration over ξ may be accomplished analytically, as shown in the appendix, [Eq. A5], that is,

$$I = \int_0^\infty d\xi \cos v_z \xi \cdot p_3 = \frac{1}{(2\pi\sqrt{M_{11}})} e^{-c} \cos v_z B e^{-v_z^2/4A} \tag{8}$$

where

$$A = M_{11}/(2|R|)$$

$$B = M_{12}/M_{11})(\mu_1 + \mu_2)$$

$$c = (1/2|R|)\,[(M_{22} - M_{12}^2/M_{11})(\mu_1^2 + \mu_2^2) + 2(M_{23} - M_{12}^2/M_{11})\mu_1\mu_2]$$

M_{ij} is the co-factor of the covariance matrix R_{ij}. One may rewrite the quantity c as

$$c = (\sigma^2/T^2)(1/M_{11})[\bar{E}(\mu_1^2 + \mu_2^2) + 2\bar{F}\mu_1\mu_2]$$

where

$$\bar{E} = (1/(\sigma^2/T^2)\,((M_{11}M_{22} - M_{12}^2)/(2|R|))$$

and

$$\bar{F} = (1/(\sigma^2/T^2)\ ((M_{11}M_{23} - M_{12}^2)/(2|R|))$$

After considerable algebraic manipulation, one can show that $\bar{E} = 1$ and

$$\bar{F} = -\exp(-\tau^2/T^2)(1 - 2\tau^2/T^4)$$

$$1 - \bar{F}^2 = M_{11}/(4\sigma^4/T^4)$$

By completing the square, one may rewrite c as

$$c = (\sigma^2/T^2)(1/M_{11})\left[(\mu_1^2 + \mu_2^2 + 2\bar{F}\mu_1\mu_2 + \bar{F}\mu_1^2) - \bar{F}^2\mu_1^2\right] \tag{9}$$

Now, define $\mu_2' = \mu_2 + \bar{F}\mu_1$, so that c becomes

$$c = \left[(\frac{\sigma^2}{T_2})(\frac{\mu_2'^2}{M_{11}}) + \frac{\mu_1^2}{4(\sigma^2/T^2)}\right] \tag{10}$$

The question arises, what happens to the expression for I given by [Eq. (8)] as $\tau \to 0$ $(x_1 \to x_2)$. It can be shown that $M_{11} \to (\frac{24\sigma^4}{T^4})(\tau^2/T^2)$ and $|R| \xrightarrow[\tau \to 0]{} (\frac{8\sigma^6}{T^4})\ (\tau^8/T^8)$. Also, as $\tau \to 0$, $\xi \to 0$, $\bar{F} \to -1$ and $\mu_1 \to \mu_2$ so that $\mu_2' \to 0$. Then we have

$$c \xrightarrow[\tau \to 0]{} [c_1 + \mu_1^2/4(\sigma^2/4(\sigma^2/T^2))],$$

where c_1 is a constant.
So that if

$$\mu_1 = 0,\ c \xrightarrow[\tau \to 0]{} c_1$$

and

$$I \xrightarrow[\tau \to 0]{} (1/(2\pi|\tau|)e^{-c_1}$$

Hence, I and therefore $w(\tau)$ has a singularity at $\tau = 0$. However, it can be shown that this singularity is integrable. This may be seen from [Eq. (5)]. Here, the behavior of $W(\tau)$ near $\tau = 0$ is governed by $G(\tau)$, which in turn, is governed by the behavior of P_3 near $\tau = 0$. But, from [Eq. (A3)] in the appendix,

$$P_3 = (2\pi)^{-3/2}\ |R|^{-1/2}e^{-Q}$$

where

$$Q = \underset{\sim}{u}^T\ \underset{\sim}{R}^{-1}\underset{\sim}{u}.$$

Now,

$$|R| \xrightarrow[\tau \to 0]{} (8\sigma^6/T^4)(\tau^8/T^8)$$

and

$$Q \xrightarrow[\tau \to 0]{} 1/(c_2\tau^8)[c_3\tau^2\xi^2 + c_4\tau^3\xi(\mu_1 + \mu_2),$$

where c_2, c_3, and c_4 are constants.
Therefore, since $\xi \to 0$ and $\mu_1 \to \mu_2$, $Q \xrightarrow[\tau \to 0]{} + \infty$
so that

$$P_3 \xrightarrow[\tau \to 0]{} 0$$

and $W(\tau)$ is fininte as $\tau \to 0$. Thus, $W(\tau)$ is integrable about the point $\tau = 0$. How this may be accomplished numerically is explained in Section 4.

4. NUMERICAL EVALUATION OF NORMALIZED CROSS SECTION

This section discusses the numerical evaluation of the simplified expression for the normalized cross section. We address round off errors, integration schemes and limits of integration. Numerical integration of large complicated functions is always challenging, but especially in our case. Using $\mu_2' = \mu_2 + \bar{F}\mu_1$, we have the cross section

$$\sigma^\circ = 2\pi k \int_0^\infty d\tau \cos(v_x\tau)\ W(\tau)$$

where

$$W(\tau) = G(\tau) - H$$

and

$$G(\tau) = \int_{-\infty}^{\infty} d\mu_1 \int_{-\infty}^{\infty} d\mu_2' \left[\bar{F}(\mu_2' - \bar{F}\mu_1)F(\mu_1) \cdot e^{-c} \cos(v_z B) \left(1/\pi\sqrt{M_{11}}\ e^{-v_z^2/4A} \right) \right]$$

$$A = M_{11}/(2|R|)$$

$$B = (M_{21}/M_{11})(\mu_2' + (1 - \bar{F})\mu_1)$$

$$c = (\sigma^2/T^2)(\mu_2'^2/M_{11}) + \mu_1^2/(4(\sigma^2/T^2))$$

The function oscillates and is subject to severe round-off error. When the common factors in the numerator and denominator B and A are eliminated we have

$$B = -\tau/(1 - 2\tau^2/T^2 + e^{-\tau^2/T^2})$$

and

$$A = (1/_4\sigma^2)\left[1 + \frac{(1 + e^{-\tau^2/T^2})}{\alpha - \beta} \right]$$

where

$$\alpha = (1 - \tau^2/T^2 + {}^1\!/_2\tau^4/T^4 - e^{-\tau^2/T^2}) \xrightarrow[\tau \to 0]{} -{}^1\!/_6\tau^6/T^6$$

$$\beta = (1 + \tau^2/T^2 + {}^1\!/_2\tau^4/T^4 - e^{+\tau^2/T^2}) \xrightarrow[\tau \to 0]{} {}^1\!/_6\tau^6/T^6$$

Note the first three elements of α and β are the first three terms of the Taylor series expansion of the exponential. Let $x = -\frac{\tau^2}{T^2}$ for α and $x = +\frac{\tau^2}{T^2}$ for β. The Taylor series expression for e^x is

$$e^x = 1 + x + x^2/2! + \ldots. = \sum_{h=0}^{\alpha} \frac{x^h}{h!}$$

so $1 - e^x = -\sum_{h=1}^{\alpha} \frac{x^h}{h!}$

$$1 + x + \frac{x^2}{2} - e^x = -\sum_{h=3}^{\infty} \frac{x^h}{h!}$$

if $|x|$ is made small $e^x = 1$. For example if $x = 0.01$ then

$$e^{-0.01} = 1 - 0.01 + \frac{10^{-4}}{2} - \frac{10^{-6}}{6} + \ldots$$

Many or all of the significant figures could be lost by finding e^x then subtracting the first few terms. So the program was written to find $(1 - e^x)$ and $(1 + x + x^2/2 - e^x)$ directly by the sum series for small $|x|$. $\sum_{h=1}^{\infty} \frac{x^h}{h!}$ was used for M_{11} where

$$M_{11} = 4\frac{\sigma^4}{T^4}\left[(1 - e^{-2\tau^2/T^2}) + 4e^{-2\tau^2/T^2}\left(\frac{\tau^2}{T^2} - \frac{\tau^4}{T^4}\right)\right] \xrightarrow[\tau \to 0]{} 24\frac{\sigma^4}{T^4}(\tau^2/T^2) \qquad (12)$$

4.1 Limits of Integration

The function drops off exponentially in μ_1 and μ_2' so we use this property to determine the practical limits of integration. First we integrate [Eq. (A1)] to get H. The exponential part is

$$\exp - \left[\mu/(2\sigma/\tau)\right]^2$$

The function drops off by $e^{-16} \approx 10^{-7}$ when $\mu_{max} = + {}^{8\sigma}/\tau$

As

$$t \to \infty,\ G(\tau) \to H \text{ so } w(\tau) = G(\tau) - H \to 0$$

In some cases H is extremely small so it can be neglected and the behavior of only G need be considered. For single integral physical optics (PO) we use an upper limit of $\tau_{max} = 4T$. At that point the difference has dropped off by $e^{-16} \approx 10^{-7}$.

The full triple integral for physical optics (TI) is a very complex function in τ. Therefore the limits cannot be determined analytically. However, we do expect a limit comparable to the PO integration limit. We find the value of $w(\tau)$ for

$$\tau = n\frac{T}{2} \text{ for } n = 1, 2, \ldots\ldots 20$$

The first point where two consecutive values of $w(\tau)$ drop below 10^{-10} is taken as τ_{max}, the upper limit of integration. Two consecutive $w(\tau_n)$ are checked in case one is a null caused by the function's oscillations. Our experience showed this method produces reasonable values for τ_{max}.

Next we find the limits in μ_1 and μ_2'. The exponential part of the expression is e^{-c} where

$$c = \frac{1}{M_{11}}\left(\frac{\sigma\,\mu_2'}{T}\right)^2 + (\,\mu_1/(2\sigma/\tau))^2$$

Good limits are

$$\mu_{1max} = \pm\,\sigma/T$$

$$\mu_2' = \pm\,4\sqrt{M_{11}}/(\sigma/T)$$

Either will cause c to increase by 16 which causes the function to drop off by $e^{-16} \approx 10^{-7}$. Note that μ'_{2max} changes with τ sinced M_{11} is a function of τ. When τ is small M_{11} and μ'_{2max} are small so the function is integrated over a narrow range. When τ is large the range of integration in μ_2' is broader. The flexible integration range cuts down on wasted computational effort at small τ.

4.2 Integration Procedure

The next step is to pick an integration scheme. Common integration schemes are Simpson's rule and Romberg integration which is a higher order variation on Simpson's rule. Unfortunately, the function oscillates and the results diverge even with more than 16,000 points and days of CPU. This occurs because Simpson's rule acts like a digital filter amplifying the large high order frequencies. At finer step sizes, the integrated sum will grow in size for a high-frequency function (Hamming[10], p. 39).

Another common integration scheme is Gaussian quadrature. Plots of the function in μ_1 and μ_2' show that it has about 10 to 20 cycles of oscillation in the interval of integration. So each integrand was divided up into 20 sections and each section was solved by Gauss-Legendre quadrature and the sections were totaled up to give the integral for that integrand.

The integration is done in a very complex iterative method. Before integrating, the abscissas and weights are determined for Gauss-Legendre integration with 2, 4, 8, 16, 32, and 64 points. Each section was first found by 2 point quadrature, then with 4 point, then 8 point, etc. After each step the sections and their sum are compared to see if the results have converged. The sections which converge or fall below the error threshold are marked "done". When the total of the sections converges we have the solution for that integral. This is done for all three levels of integration.

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10. Hamming, R.W. (1977) *Digital Filters*, Prentice Hall, New Jersey.

In the outer two integrations (over τ and μ_1), an error threshold is determined which is passed to the next inner integration (over μ_1 and μ_2'). If a section or the sum falls below this threshold, the respective portion of the inner integration is marked "done" even if it has not converged, because the value is too small to significantly contribute to the final result. The complicated iterative method saves computer time, but greatly increases the program complexity.

Table 1 shows the subroutine calls needed to perform the actual triple integration. The program is ten subroutines deep. MAIN is the main program. It calls subroutine QUADT. Subroutines QUADT, QUADM1, and QUADM2 set up the parameters for the quadrature over τ, μ_1 and μ_2' respectively. The QUAD subroutines call subroutines IG, IGM1, and IGM2 respectively. These subroutines do the actual integrations of each section of the integrand over τ, μ_1, and μ_2'. The IG subroutines respectively call WCOS, R, and P, which find the functions in τ, μ_1, and μ_2'. WCOS and R call the next inner-most quadrature subroutines (QUADM1 and QUADM2). Subroutine WCOS also calls S1 and S3 which find $(1 - e^x)$ and $(1 + x + x^2/2 - e^x)$ respectively. Subroutines R and P call F which finds $F(\mu)$.

Table 1. Calculation of Normalized Cross Section $\sigma°$

Geometric Optics	Physical Optics	Triple Integral
MAIN	MAIN	MAIN
F	QUAD	QUADT
	IG	IG
	FNPO	S3–WCOS–S1
		QUADM1
		IGM1
		R–F
		QUADM2
		IGM2
		P–F

5. NUMERICAL RESULTS

In this section, graphs of $\sigma°$ will be presented as a function of the Rayleigh roughness parameter Σ for different slope conditions, σ/T. For a one dimensional rough surface, three models are used to represent the normalized cross section, $\sigma°$: (1) The triple integral representation (TI) given by [Eqs. (4), (5), A1 and A5,] (2) the single integral representation (PO) given by [Eq. (6)], valid for small slopes and (3) the asymptotic, high frequency representation (GO), given by [Eq. (7)]. In this report, the parameters used to generate the figures were taken to have the following values: λ ranges from 0.01 m to 0.35 m,

$\phi_s = 0°$ and the complex dielectric constant of the surface $\varepsilon = 4.0$. The Rayleigh parameter was varied by varying the em wave-length λ. The angles of incidence and scattering are always fixed at 89.75°.

In Figure 1, the mean slope $\sigma/T = 0.1$, so that the PO single integral representation is valid. It can be seen that the PO representation of $\sigma°$ is within 0.1 percent of the exact TI representation for all Rayleigh parameters Σ. The high frequency GO representation is within 0.5 percent of the TI representation only for $\Sigma > 5$. For $\Sigma < 2$, the GO representation is completely inaccurate.

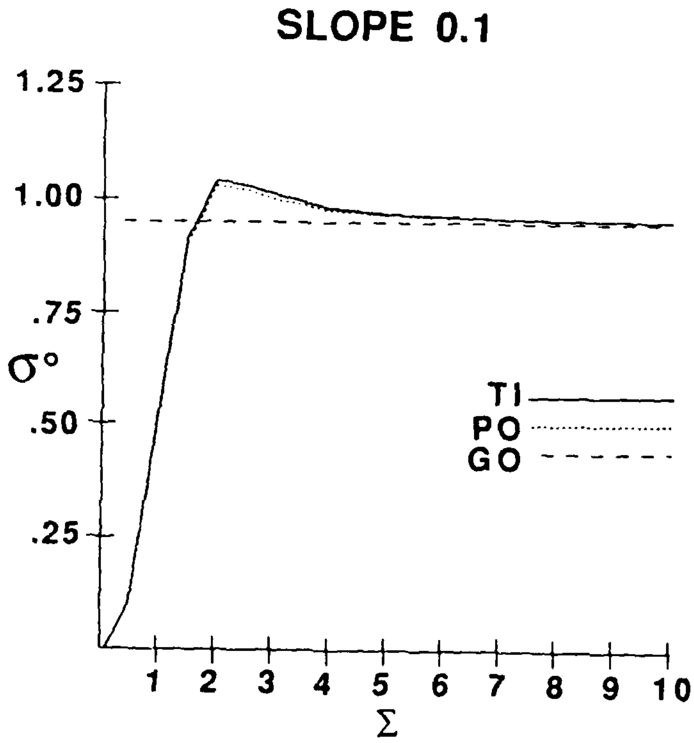

Figure 1. Normalized Cross Section $\sigma°$ vs. Rayleigh Parameter Σ, $\sigma/T = 0.1$

In Figure 2, the mean slope σ/T has been increased to 0.2. Now, there is a greater discrepancy between the PO representation and the TI representation, but the PO representation is still within 0.5 percent of the TI representation for all values of Σ. The GO representation is inaccurate for $\Sigma < 4$, compared to the TI representation.

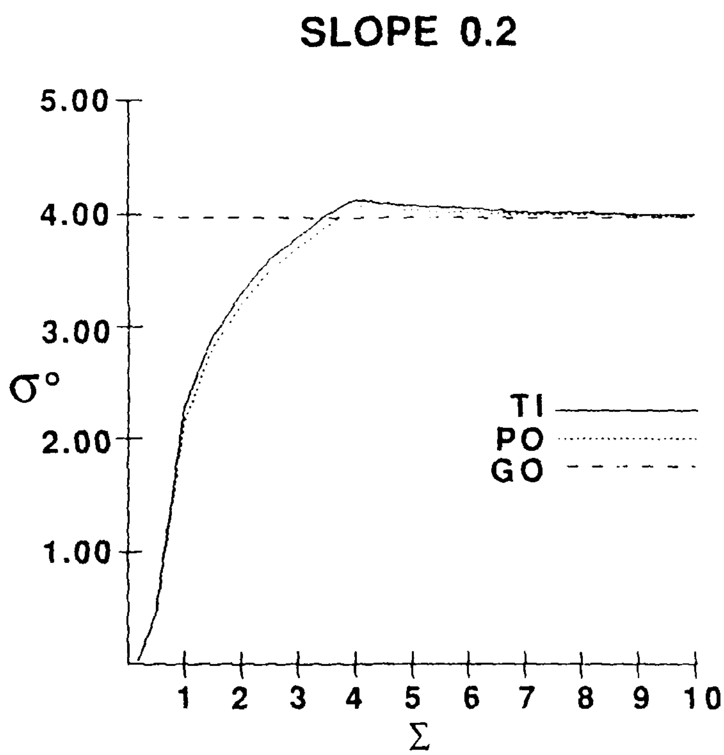

Figure 2. Normalized Cross Section σ° vs. Rayleigh Parameter Σ, $\sigma/T = 0.2$

In Figure 3, the mean slope has been increased to 0.5. Here, the discrepancy between the PO representation and the TI has increased to about 5 percent for $\Sigma > 5$ and is as large as 20 percent in the vicinity of $\Sigma \approx 2$. One can infer that the single integral PO representation is accurate only for slopes $\sigma/T < 0.5$. The GO representation is within 5 percent of the accurate TI representation only for $\Sigma > 5$.

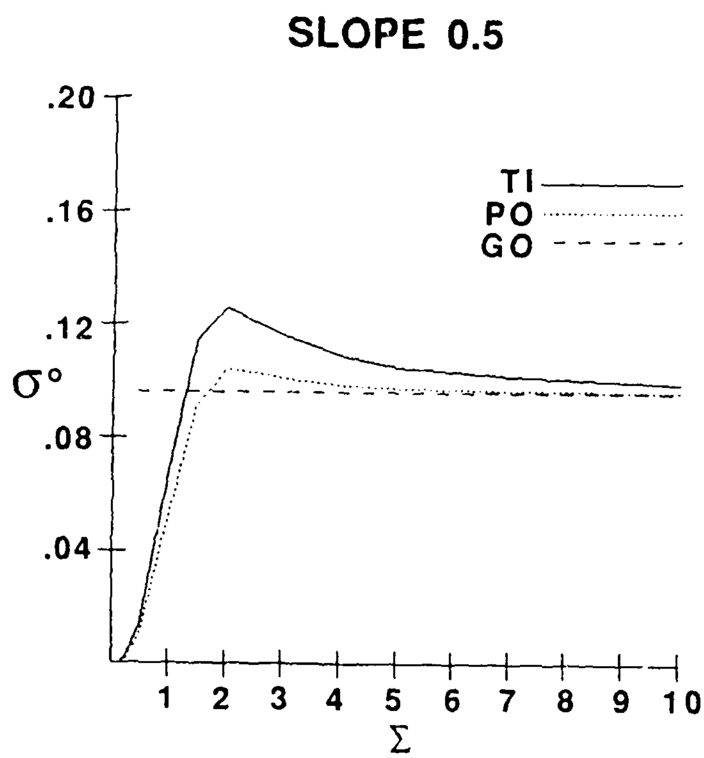

Figure 3. Normalized Cross Section $\sigma°$ vs. Rayleigh Parameter Σ, $\sigma/T = 0.5$

In Figure 4, the mean slope has increased to 1.0. The discrepancy between the PO representation and the TI is 20 percent for $\Sigma > 5$ and is as large as 45 percent near $\Sigma \approx 3$. Thus, the single integral PO representation is not valid or accurate for $\sigma/T > 0.5$. Also, it should be noted that the high frequency GO representation is not accurate for slopes $\sigma/T > 0.5$, regardless of the value of the Rayleigh parameter Σ.

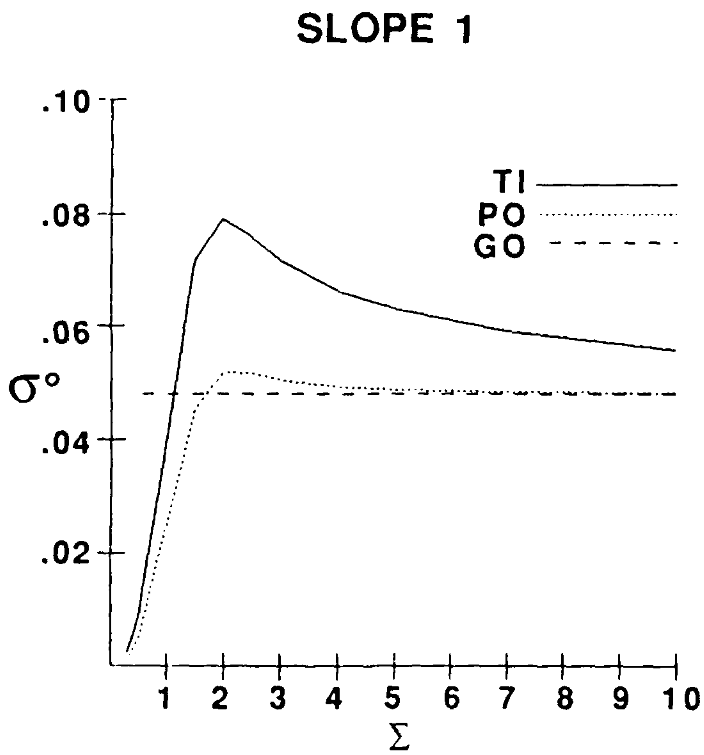

Figure 4. Normalized Cross Section σ° vs. Rayleigh Parameter Σ, $\sigma/T = 1.0$

In Figure 5, the main slope has been increased to 2.0. Here, the discrepancy between the single integral PO model and the TI is 100 percent for a range of Σ values, so that neither the PO model nor the GO model are accurate for most Σ values.

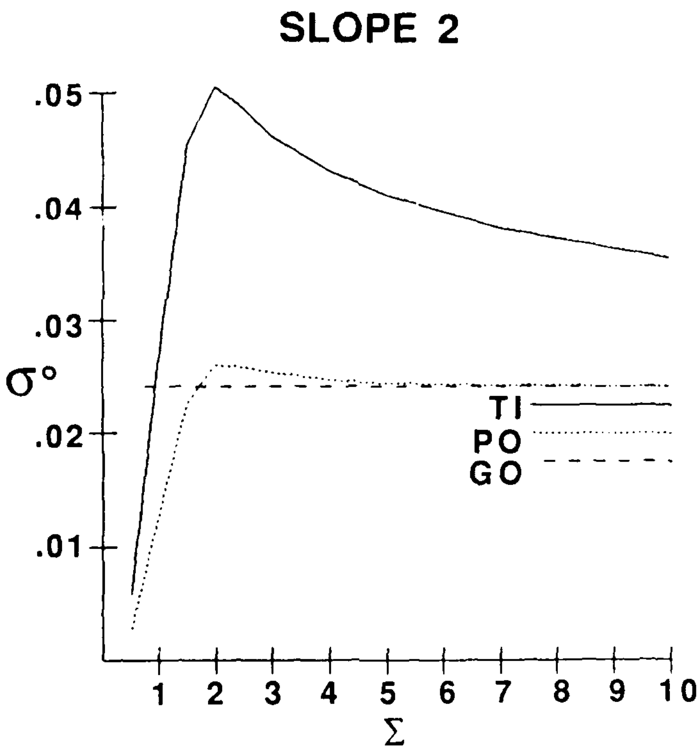

Figure 5. Normalized Cross Section $\sigma°$ vs. Rayleigh Parameter Σ, $\sigma/T = 2.0$

In Figure 6, the mean slope has been further increased 5.0. Now, the slope is so large that neither the PO model nor the GO model are accurate for most Σ values.

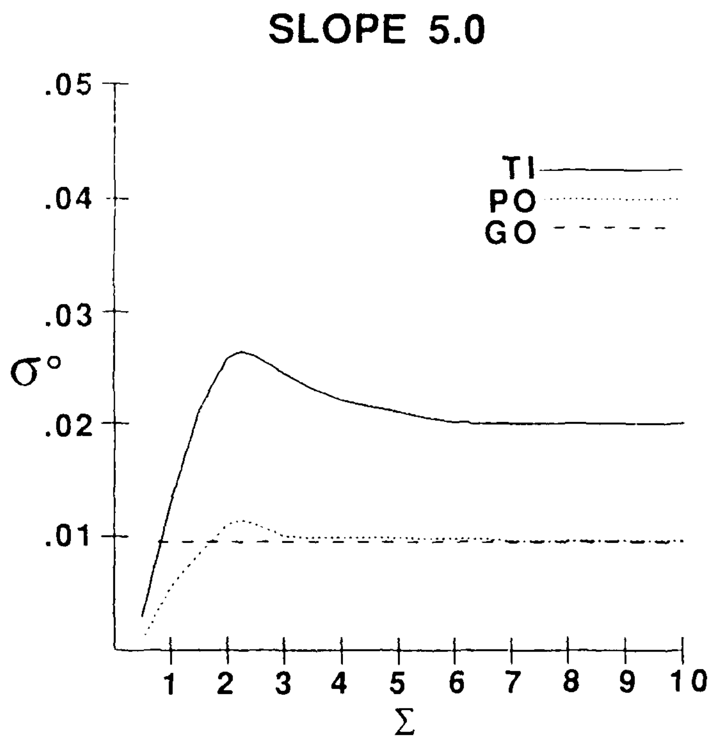

Figure 6. Normalized Cross Section σ° vs. Rayleigh Parameter Σ, $\sigma/T = 5.0$

In Figure 7, plots are shown of Δ (in percent) vs σ/T for different Rayleigh parameter Σ regimes, that is, for different frequency ranges. Here, the percent error

$$\Delta = \frac{\sigma^\circ(TI) - \sigma^\circ(PO)}{\sigma^\circ(PO)}.$$

For $\Sigma < 1$ we took $\Sigma = 1/2$, for $1 < \Sigma < 4$ we took $\Sigma \approx 3$, and for $\Sigma > 4$ we took $\Sigma \approx 6$. It may be noted that the greatest error lies in the Rayleigh parameter regime $1 < \Sigma < 4$. Also, the error increases as the slope σ/T increases. If one calculates according to the single integral (PO) representation, one can recover the exact TI σ° value by multiplying the σ°(PO) value by the appropriate Δ value and adding it to the (PO) value, that is

$$\sigma^\circ(TI) = \sigma^\circ(PO) + \Delta[\sigma^\circ(PO)].$$

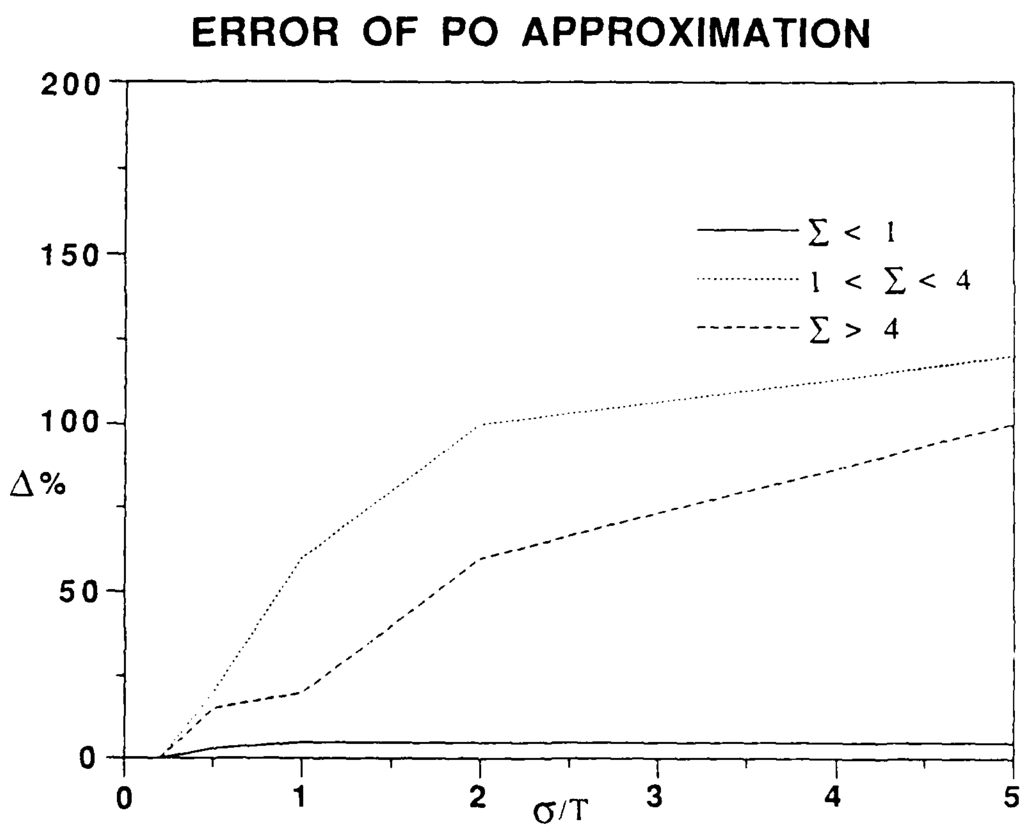

Figure 7. Relative Error vs. Slope for Different Rayleigh Parameters

6. DISCUSSION

In this report, it was shown that an exact representation of a physical optics model for the normalized cross section σ° could be derived for a one-dimensional rough surface. This exact representation of σ° was reduced to a triple integral over the separation τ and the surface slopes at two points, μ_1 and μ_2. It was demonstrated that the integral over τ had a singularity, but that it was integrable. The integrals could be performed numerically. The Romberg method of numerical integration did not converge; a Gaussian quadrature technique had to be employed. Three models for σ°, the triple integral (TI), single integral physical optics (PO), and the geometrical optics (GO) were studied as a function of Rayleigh parameter Σ for different slope (σ/T) regimes. The Rayleigh parameter was varied by varying the frequency; the angles of incidence θ_i and scattering θ_s are fixed at 89.75°. A graph of relative error Δ vs σ/T for different Rayleigh parameter Σ regimes was presented. It can be used to recover the exact σ° (TI) value from the single integral PO representation of σ°, for this set of conditions.

7. CONCLUSIONS

It was found that the PO model is accurate only for $\sigma/T < 0.5$, regardless of the Rayleigh parameter. It was also found that taking the high frequency limit $\lambda \rightarrow 0$ is not a sufficient condition for the validity of the GO model. From the numerical results, it was shown that the surface slopes must be small; $\sigma/T < 0.5$. From analysis of the integral representation of σ°, it can be shown that the asymptotic expression (GO) for σ° may be derived if $\Sigma > 4$.

References

1. Papa, R.J. and Lennon, J.F. (1988) Regions of Validity for Some Rough Surface Scattering Models, "Scattering and Propagation in Random Media", AGARD Conference Proceedings, 419.
2. Papa, R.J., Lennon, J.F., and Taylor, R.L. (1984) *An Analysis of Physical Optics Models for Rough Surface Scattering*, RADC-TR-84-195, ADA154960.
3. Barrick, D.E. and Peake, W.H. (1967) *Scattering from Surfaces with Different Roughness Scales: Analysis and Interpretation*, Battelle Report, AD662751.
4. Semyonov, B.I. (1966) Approximate computation of scattering of electromagnetic waves by rough surface contours, *Radiotekhnika i Electronika* **11**, 1179–1187.
5. Barrick, D.E. (1968) Rough surface scattering based on the specular point theory, *IEEE Trans. Antennas Propag.* **AP-16**(4), 449–454.
6. Kodis, R. (1966) A note on the theory of scattering from an irregular surface, *IEEE Trans. Antennas Propag.* **AP-14**(1), 77–82.
7. Sancer, M.I. (1969) Shadow corrected electromagnetic scattering from a randomly rough surface, *IEEE Trans. Antennas Propag.* **AP-17**(5), 575–585.
8. Ruck, G.T., Barrick, D.E., Stuart, W.D., and Krichbaum, C.K. (1970) *Radar Cross Section Handbook, Vol.* **2**, New York, Plenum Press.
9. Hagfors, T. (1964) Backscattering from an undulating surface with applications to radar returns from the moon, *J. Geophysical Res.* **69**(18), 3779–3784.
10. Hamming, R.W. (1977) *Digital Filters*, Prentice Hall, New Jersey.

Appendix

The Relationships in the General Integral Expression for Cross Section

The purpose of this appendix is to give the details of the respective arguments by which the results of [Eq. (4)] and [Eq. (5)] can be simplified to those of [Eq. (6)] for both sets of assumptions. The first step is to examine the elements of the integrands in more detail.

First, the quantity H used in $w(\tau)$ in [Eq. (4)] is given by the expression

$$H = \left(T^2 e^{-v_z^2\sigma^2}/(4\pi\sigma^2)\right) \left[\int_{-\infty}^{\infty} \exp - \left(\mu/(2\sigma/T)\right)^2 F(\mu)\,d\mu\right]^2 \tag{A 1}$$

The function $F(\mu)$ used in [Eq. (5)] can be reduced to the form

$$F(\mu) = (1 - R_\perp(\gamma_i))(\mu\sin\theta_i + \cos\theta_i) + [1 + R_\perp(\gamma_i)]\,(\mu\sin\theta_s - \cos\theta_s) \tag{A 2}$$

where it has been assumed that there is no y variation ($\frac{\partial}{\partial y} = 0$) and the scattering takes place in the forward direction ($\varnothing_s = 0°$). Here, the Fresnel reflection coefficient is given by

$$R_\perp(\gamma_i) = (\cos\gamma_i - \sqrt{\varepsilon - \sin^2\gamma_i})/(\cos\gamma_i + \sqrt{\varepsilon - \sin^2\gamma_i})$$

and

$$\cos\gamma_i = (\mu\sin\theta_i + \cos\theta_i)/(\sqrt{1 + 2\mu^2}).$$

The trivariate distribution function P_3 in [Eq. (5)] is given by

$$P_3 = (2\pi)^{-3/2}\,|R|^{-1/2}\,e^{-\underset{\sim}{u}^T \underset{\sim}{R}^{-1} \underset{\sim}{u}}. \tag{A 3}$$

$\underset{\sim}{u}^T\ \underset{\sim}{R}^{-1}\underset{\sim}{u}$ is a positive definite quadratic form:

$$\underset{\sim}{u}^T \underset{\sim}{R}^{-1}\underset{\sim}{u} = 1/(2|\underset{\sim}{R}|)[M_{11}\,\xi^2 + 2M_{12}\,\xi(\mu_1+\mu_2) + M_{22}\,(\mu_1^2+\mu_2^2) + 2M_{23}\mu_1\mu_2]$$

$$\underset{\sim}{u}^T = (\xi, \mu_1, \mu_2)$$

$|R|$ is the determinant of the surface height covariance matrix

$$\underset{\approx}{R} = \begin{pmatrix} 2(\sigma^2-\rho) & -\partial\rho/\partial\tau & -\partial\rho/\partial\tau \\ -\partial\rho/\partial\tau & +\partial^2\rho/\partial\tau^2\ |_{\tau=0} & -\partial^2\rho/\partial\tau^2 \\ -\partial\rho/\partial\tau & -\partial^2\rho/\partial\tau^2 & +\partial^2\rho/\partial\tau^2\ |_{\tau=0} \end{pmatrix} \tag{A4}$$

where

ρ = surface correlation function

$$\rho = \sigma^2\exp(-\tau^2/T^2)$$

It should be noted that the expression for $\underset{\approx}{R}$ given by Hagfors[9] has errors in the signs of several elements.

M_{ij} is the co-factor of the covariance matrix R_{ij}. The triple integral in [Eq. (5)] may be reduced to a double integral when $\tau \neq 0$ by using the known expression for the cosine transform of a Gaussian function

$$\int_0^\infty d\xi \cos v_z\,\xi P_3 = 1/(2\pi\sqrt{M_{11}})e^{-C}\cos(v_z B)\exp(-v_z^2/4A) \tag{A5}$$

where

$$A = M_{11}/(2|R|)$$

$$B = (M_{12}/M_{11})(\mu_1+\mu_2)$$

$$C = (1/2|R|)\left[(M_{22} - M_{12}^2/M_{11})(\mu_1^2+\mu_2^2) + 2(M_{23} - M_{12}^2/M_{11})\mu_1\mu_2\right]$$

At this point we turn to the two cases that are derived under different assumptions.

First, consider the small slope case, $\sigma/T \ll 1$. In this case, the covariance matrix $\underset{\approx}{R}$ becomes

$$\underset{\approx}{R} = \begin{pmatrix} 2(\sigma^2-\rho) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{A6}$$

so that the heights and slopes are decorrelated. Then the integrations over μ_1 am μ_2 in [Eq. (5)] may be approximated by setting up $\mu_1 = \mu_2$ = constant = $\tan\gamma = \mu_{sp}$ where $\tan\gamma$ is the slope of a facet which will produce a reflected wave in the specular direction:

$$\tan\gamma = \left| \sin\theta_i - \sin\theta_s \right| / (\cos_i + \cos\theta_s) \quad (A7)$$

[Eq. (4)] for σ° now reduces to a single integration form as shown in [Eq. (6)]. It should be noted that since the slopes are assumed to be small ($\sigma/T << 1$), then $\mu = \tan\gamma$ so that $\gamma \approx 0$, which implies the specular condition $\theta_s = \theta_i$.

The argument for reducing the integrals in [Eq. (5)] to the result in [Eq. (6)] is somewhat different for the high frequency case. In the GO limit the integrals over μ_1 and μ_2 may be approximated by removing $F(\mu_1)$ and $F(\mu_2)$ from the integrals and setting them equal to constants; the justification for this is the stationary phase (or specular point) argument. This argument states that for large k in the exponential (or cosine) factor, the only surface regions which contribute to the integral are those smoothly curving portions in a position to specularly reflect into the desired scattering direction. Then, $\mu_1 \approx \mu_2 = \tan\gamma = \mu_{sp}$ and [Eqs. (4) and (5)] reduce to [Eq (6)] (see Barrick and Peake (3)).

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